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Analytical and Numerical Methods for Determining Shear Stress in a Cantilever Beam

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Abstract

The shear stress of a cantilever beam three meters long under a concentrated load at its free end is determined. In this investigation, we examine the maximum shear stresses in the rectangle (R), the inverted triangle (I), and the transverse tee using both the traditional analytical equation developed by Collingnon and the finite element method (FEM) software. Both ANSYS and SAP2000 were required for this task. There is a discrepancy between the maximum shear stresses determined by an analytical equation and those determined by a computer program. Average differences between ANSYS and SAP2000 were 12.76 and 11.96 percent, respectively, independent of the cross-section employed to determine them. These deviations necessitated the integration of cross-sectional corrective factors into the traditional analytical formula. After correction, the average deviations drop to 1.48 percent and 4.86 percent, respectively, regardless of the cross section type.Finite Element Methods; Analytical Equations; Comparison Analysis; Correction Factor are all terms associated with this paper.

Introduction

Beams have found widespread use in many fields, including construction, transportation, chemical, aerospace, and marine engineering [1]. The beam's primary structural function is to support forces that act at right angles to its longitudinal axis. There are two forces, shear and bending moment, acting on the beam's cross section when it is sheared or bent. Many beginning courses in materials and structural mechanics include stresses in beam structures.

There is a lot of nuance involved in researching beams due to the fact that stresses and moments may vary over the length of a loaded beam. Shear stresses are generated by axial forces and bending moments, while normal stresses are provided by bending moments and axial forces [2]. Internal forces acting on the transverse part of the element cause both types of strains. When calculating shear stresses, the fundamental analytical equation developed by Collignon [3] is often utilized (). This is represented by (Equation 1). This equation may be used to represent prisms made from a homogenous material if they display linear elastic behavior and the internal resultant shear force acts in a direction perpendicular to the cross-sectional area [2]. The shear force (V), the initial moment of area and the moment of inertia, t and I, of the cross sectional area relative to the neutral axis, and Collignon's formula are all used to get the shear stress (Q).

$$r = \frac{V \cdot Q}{I \cdot t}$$

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Shear stress was first calculated in the late 19th century by Collingnon. When computer sciences advanced and finite element techniques (FEM) were used in structural analysis, this formula had to be re-examined. It is possible to accurately solve complicated engineering issues using the FEM numerical approach [4]. An FEM model may quickly discover the combination of material attributes or the size of pieces that best suit a structure's needs, depending on specified criteria. Utilizing FEM-based calculations, current design models are able to take into account all of a design's inherent flaws, which are not evaluated when using analytical formulae.

Analytical equations and the finite element technique (FEM) are used to evaluate the stresses in a structure. Depending on the approach used, the results may differ. It's important to recognise this variance since it's possible that both techniques will be employed at the same time while designing the same structure. FEM and analytical formulae have been utilised to compare the disparities in shear stress measurements. When a cantilever beam with a focused load at its free end was evaluated using FEM for shear stress [6], the results were impressive. The stresses were estimated for various components, such as beams, shells, planes, and solids, using FEM software, ANSYS (American computer-aided engineering software). Analytical and modelled solutions were found to vary for each element. Using the element solid as an example, these disparities reached as high as 158.27 percent, with the FEM-derived values consistently outperforming the analytical calculation. ANSYS and the Collingnon equation were also used to investigate the maximum shear stresses in a beam [7]. ANSYS's results indicated discrepancies of up to 10%, which was larger than the analytical method's. According to these experts, engineers may build a safer design by taking into account the stresses that can be derived from the FEM.

It is to be anticipated that there may be discrepancies in calculations among the several FEM software packages available. No literature has been found on the subject of SAP2000, a FEM engineering simulation programme that is widely used in civil engineering projects.

Methodology

ANSYS 8.0 and SAP2000 were used to create three-dimensional linear finite element models of concrete beams. Using a prismatic concrete cantilever beam that was subjected to a certain load on the free end, the maximum shear stress was determined. An investigation of how geometry influences the outcomes was conducted using three cross sections (rectangular, I, and T).

Formula 1 (Equation 1) was compared with the numerical results of FEM, and the percentage differences were also determined. Corrected classical equations were presented based on a comparison between analytic methods and FEM models. The fit of corrected equations was evaluated based on this comparison.

Case Study

Definition of the Structural Element

A three-meter cantilever exposed to a punctual load (P) at its free end is used to simplify the case study (Figure 1a). Beam weight is omitted in order to isolate the impact of shear force on cross section without the influence of other forces.. This scenario has been simplified by disregarding the torsion moments and the axial forces, while keeping the shear force diagram constant over the cantilever's whole length (Figure 1b).

The most common concrete cross sections [8] were examined: rectangular (R), octagonal (I), and triangular (T). Figure 1c shows the sectional geometry.

Maximum Shear Stresses Assessment

Analytical Equation

The Collignon formula was used to compute the maximum shear forces for each cross section region. Equation 1 is computed using the punctual load P as the sole unknown. The R cross section situation is used as an example.

With respect to the shear force diagram shown in Figure 1b, the shear force (V) is exactly equal to the punctual load (P). The width (t) was determined to be 100 mm based on the rectangle's shape. Top (or bottom) component A* is defined as A*.

Above (or below) the section plane where is (t) is measured, the cross-sectional area of the members Equivalent to the distance between the centroid of (A) and the neutral axis (y'), the cross-sectional moment of inertia (I) and the first moment of area (Q) were derived using Equations 2 and 3, respectively.

$$I = \frac{bh^3}{12}$$

$$Q = A^{*}.\, y' = b.\, \frac{n}{2}.\, y'$$

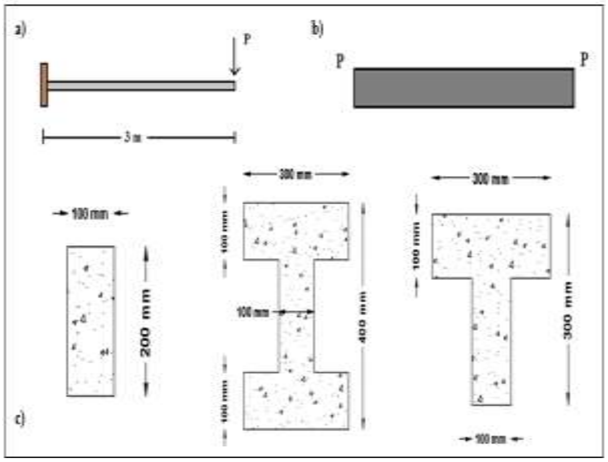


Figure 1. (a) Cantilever; (b) shear force diagram; (C) Cross-section areas

Then, Equation 4 defines the maximum shear stress (r_{max}) for the cantilever with R cross section based on the punctual load P. Notice that P is the variable considered in this study. Equation 5 and Equation 6 measure r_{max} for the cross section I and T cases, respectively. Units of P and r_{max} are kN and MPa, respectively.

$$r_{max} = 7.5 \cdot 10^{-2} \cdot P$$

$$r_{max} = 3.4 \cdot 10^{-2} \cdot P$$

$$r_{max} = 4.9 \cdot 10^{-2} \cdot P$$

Finite Element Modeling and Material properties

In order to determine the maximum shear stresses in the investigated cantilever beams, FEM simulations were performed using ANSYS and SAP2000. It was necessary to do the following stages in order to create a cantilever beam structural model:

A 3D modeller was used to create three cantilever beams, one for each part. Since these modellers replicate genuine buildings with 3D solid parts, they are more natural [9].

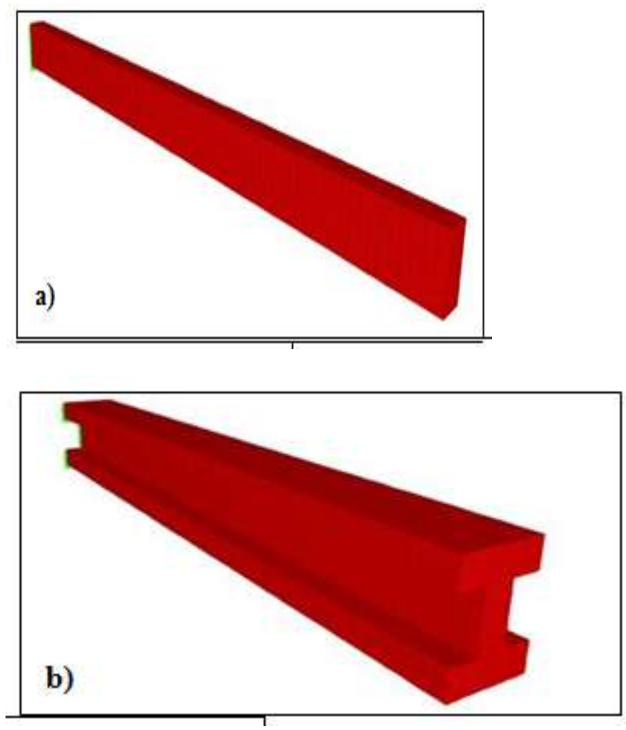
In order to keep the cantilever's fixed support from shifting, constraints were placed on it.

It was applied to all the beams models with a typical compressive strength (f') of 28 MPa and an elastic modulus (Ec) equal to 29 GPpa. Concretes with a compressive strength of 15 to 45 MPa [10] are the most common, so that's what we went with. Keep in mind that Ec was derived from Eurocode 2 [11] using the equation stated there.

• The element was subjected to meshing. There are two different FEM programmes that benefit from this mesh, as you can see. As a consequence, a finer mesh will not provide a more precise outcome (this result should me mentioned in the results). As shown in Figure 2, the beams produced using SAP2000 as an example of meshing characteristics employed in the research are shown in Table 1.

The model was subjected to the given load P. As previously stated, P is the parameter of interest in our investigation.

In the end, the model was run and results were received.



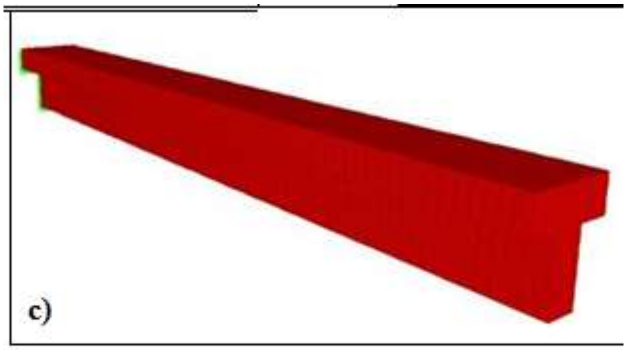


Figure 2. Meshing of the cantilever: (a) R; (b) I; (c) T crosses sections

Table1. Description of Meshing of the Cantilever

Section	Information
R	Total number of nodes: 17556 Total number of elements: 15000 The Size of each element: 40×10×10 mm
I	Total number of nodes: 67716 Total number of elements: 60000 The Size of each element: 40×10×10 mm
T	Total number of nodes: 42636 Total number of elements: 37500 The Size of each element: 40×10×10 mm

Results

Analysis Results by Analytical Equation

Table 2 presents the results of maximum shear stresses (r_{max}) measured by using the analytical equations derived for each studied cross sections. These cross sections were estimated by using Equation 4, 5 and 6 for R, I, and T cross sections, respectively. r_{max} and load values (P) equal to 50, 100, 200, 300, 400 and 500 kN.

Table 2. Maximum shear stress (r_{max}) assessed using the analytical equation (Unit: MPa)

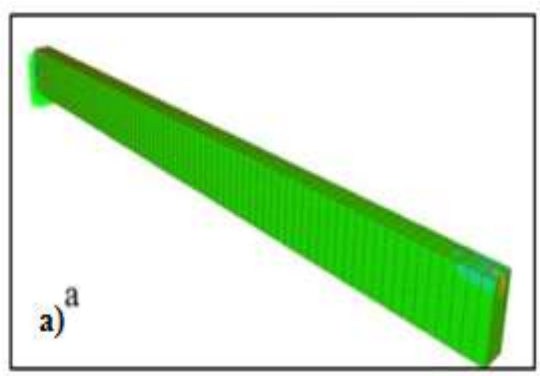
Section	P(kN)					
	50	100	200	300	400	500
R	3.75	7.50	15.00	22.50	30.00	37.50
I	1.70	3.40	6.80	10.20	13.60	17.00
T	2.50	4.99	9.98	14.97	19.96	24.95

Analyses Results by Numerical Methods

Figure 3 shows an example of the maximum shear stress calculated using SAP2000 for each of the cross sections that were examined. Keep in mind that the majority of the beams are mostly presented in a single distinct hue. Due to the continuous shear pressures and cross section of the beams, this phenomena was predicted. The fixed end of the beam and the opposite extremity, where the weight is applied, have somewhat different hues. St. Venant's principle, which involves a stress distortion of boundary conditions, is to blame for this alteration [12].

The ANSYS and SAP2000 shear stresses (rmax) are summarised in Table 3. The data is sorted according to cross section and load value (P). As predicted by the analytical equations, P used the same numbers for P's calculations.

Both ANSYS and SAP2000 provide findings that are different. For cross sections R and T, the rmax obtained with ANSYS was on average 5.63 percent and 4.88 percent greater than the rmax produced using SAP2000. When compared to SAP2000, section I's outcomes were 8.81 percent higher. However, a t-test (p-value of 0.092 > 0.05) shows that these differences are not statistically significant.



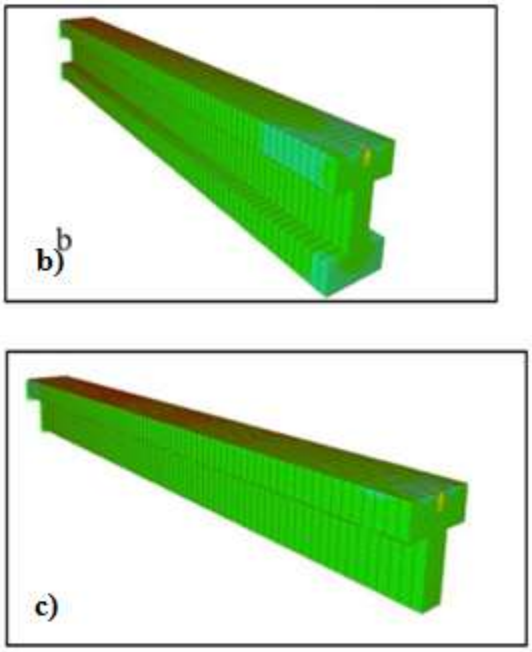


Figure 3. Shear stresses distribution: (a) R (b) I (c) T crosses sections

Table 3. Maximum shear stress (r_{max}) assessed using FEM (Unit: MPa)

Software	Section	P(kN)					
		50	100	200	300	400	500
ANSYS	R	4.39	8.79	17.59	26.39	35.19	43.99
	I	1.91	3.82	7.64	11.57	15.29	19.11
	T	2.85	5.69	11.39	17.08	22.78	28.47
SAP2000	R	4.14	8.29	16.59	24.89	33.18	41.48
	I	2.07	4.15	8.30	12.45	16.60	20.75
	T	2.71	5.42	10.84	16.26	21.68	27.11

Conclusion

Based on the results of this study, finite element approaches are preferable to conventional formulas for calculating the shear stress in a beam's cross section. This study's finding of a disparity of 14% demonstrates the need of doing follow-up research. Shear stress values for the R, T, and I regions of the beam varied by 17%, 14%, and 12% between ANSYS and SAP2000, whereas the discrepancies between the two were 11%, 9%, and 22%. By a wide margin, ANSYS and SAP2000's maximum shear stress calculations for each cross section exceeded those obtained using Collingnon's equation. This may cause shear stresses on the cross sections to be higher than those used in the analytical equations-based design, which might ultimately cause the structure to fail. Results from finite element analysis (FEA) are used to produce a safety factor for maximum shear stress, which is 1.137 regardless of the cantilever beam's cross section (Equation 7).

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